Section 4.2
Mean Value Theorem

The Mean Value Theorem for Derivatives

If $y = f(x)$ is continuous at every point of the closed interval $[a,b]$ and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Show that the function $f(x) = x^2$ satisfies the hypothesis of the Mean Value Theorem on the interval [0,2]. Then find a solution c to the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$f'(x) = 2x$

on this interval.

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = 2$$

$f'(x) = 2x$

$f'(c) = 2c$

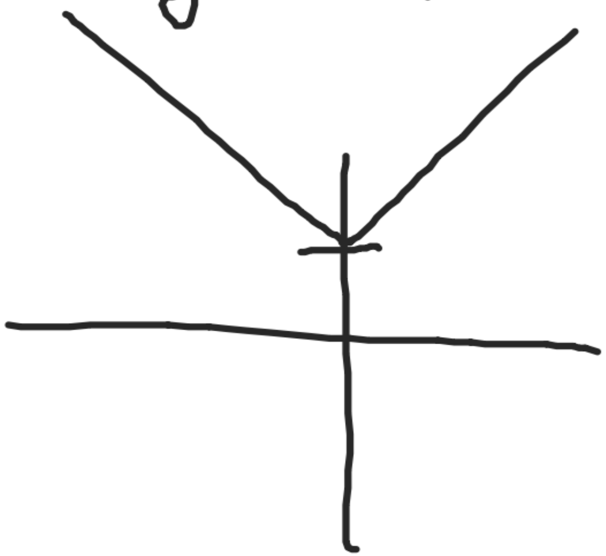
$2c = 2$

$c = 1$

Explain why each of the following functions fails to satisfy the conditions of the Mean Value Theorem on the interval [-1,1].

a.) $f(x) = \sqrt{x^2} + 1$

$y = |x| + 1$



b.) $f(x) = \begin{cases} x^3 + 3, & x < 1 \\ x^2 + 1, & x \geq 1 \end{cases}$

$f(1) = 4$

$= 2$

Show that $\frac{d}{dx}$ cos x has at least one zero in [0, 2π] using the MVT.

$$\frac{f(2\pi)-f(0)}{2\pi-0} = \frac{\cos(2\pi)-\cos(0)}{2\pi-0}$$

$$= \frac{1-1}{2\pi} = 0$$

$$-\sin x = 0$$

$$x = 2\pi$$

Let $f(x) = \sqrt{1 - x^2}$, A = (-1, f(-1)), and B = (1, f(1)).

Find a tangent to f in the interval (-1,1) that is parallel to the secant AB.

Suppose a trucker travels between two toll booths 13 miles apart in 15 minutes. The speed limit on the highway is 50 mph. Should the trucker be cited for speeding?

$$\frac{f(b)-f(a)}{b-a} = \frac{13 \text{ miles}}{15 \text{ min.}}$$

$$= \frac{13 \text{ miles}}{.25 \text{ hours}}$$

$$= 52 \text{ mph.}$$

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Increasing Function, Decreasing Function

Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1.) f increases on I if $x_1 < x_2 \rightarrow f(x_1) < f(x_2)$

2.) f decreases on I if $x_1 < x_2 \rightarrow f(x_1) > f(x_2)$

The MVT allows us to identify exactly where graphs rise and fall. Functions with positive derivatives are increasing functions. Functions with negative derivatives are decreasing functions.

Corrollary 1 to Increasing and Decreasing Functions

Let f be continous on [a,b] and differentiable on (a,b)

1.) If f ' > 0 at each point on (a,b), then f increases on [a,b]

2.) If f ' < 0 at each point of (a,b), then f decreases on [a,b]

Tell where the function is increasing or decreasing

$y = x^2$

Where is the function $f(x) = x^3 - 4x$ increasing and where is it decreasing?

Corollary 2 - Functions with f' = 0 are Constant

If $f'(x) = 0$ at each point of an interval I ,

then there is a constant C for which $f(x) = C$ for all x in I.

Corollary 3 - Functions with the Same Derivative Differ by a Constant

If $f'(x) = g'(x)$ at each point of an interval I,

then there is a constant C such that

$f(x) = g(x) + C$ for all x in I.

Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point $(0,2)$.

Antiderivative

A function F(x) is an **antiderivative** of a function f(x) if F'(x) = f(x) for all x in the domain of f.

The process of finding an antiderivative is

antidifferentiation.

Find the velocity and position functions of a body falling freely from a height of 0 meters under each of the following sets of conditions:

a.) The acceleration is 9.8 m/sec^2 and the body falls from rest.

b.) The accelearation is 9.8 m/sec^2 and the body is propelled downward with an initial velocity of 1 m/sec.